Maximal CP Violation Hypothesis
and Phase Convention of the CKM Matrix

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Abstract

The maximal CP violation hypothesis depends on the phase convention of the Cabibbo-Kobayashi-Maskawa matrix. A phase convention which leads to successful prediction under the maximal CP violation hypothesis is searched, and thereby, possible structures of the quark mass matrices are speculated.

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1 Introduction

Recent remarkable progress of the experimental B physics [1] has made possible to know the magnitude of the CP violation in the quark sector. We are interested what logic can give the observed magnitude of the CP violation. For this subject, for example, we know an attractive hypothesis, the so-called "maximal CP violation" hypothesis [2]. However, the conventional "maximal CP violation" hypothesis cannot give the observed magnitude of the CP violation, as we discuss later.

We are also interested that, which quark mass matrix element, the CP violation originates in (in other words, which of quark mass matrix elements is accompanied by a CP violating phase). However, it is usually taken that this question is meaningless, because we know that the observable quantities are invariant under the rephasing of the Cabibbo-Kobayashi-Maskawa (CKM) [3, 4] matrix. For example, we cannot physically distinguish the standard CKM matrix phase convention [5]

$$V_{SD} = R_1(\theta_{23})P_3(\delta_{13})R_2(\theta_{13})P_3^\dagger(\delta_{13})R_3(\theta_{12})$$

$$= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{13}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{13}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{13}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{1.1}$$

from the original CKM matrix phase convention by Kobayashi and Maskawa (KM) [4]

$$V_{KM} = R_1^T(\theta_2)P_3(\delta_{KM} + \pi)R_3(\theta_1)R_1(\theta_3)$$

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$$= \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta_{KM}} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta_{KM}} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta_{KM}} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta_{KM}} \end{pmatrix} , \tag{1.2}$$

where

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} , \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix} , \quad R_1(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{1.3}$$

$$P_3(\delta) = \mathrm{diag}(1, \ 1, \ e^{i\delta}), \tag{1.4}$$

$s = \sin\theta$ and $c = \cos\theta$.

Although there are many different versions of the maximal CP violation hypothesis, the conventional one demands that the nature takes a value of the CP violating phase so that the rephasing invariant quantity [6] J takes its maximal value. In the standard CKM matrix phase convention, the quantity J is given by

$$J = c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23} \sin\delta_{13} \ , \tag{1.5}$$

i.e.

$$J = \frac{|V_{11}||V_{12}||V_{33}||V_{23}||V_{13}|}{1 - |V_{13}|^2} \sin\delta_{13} \ . \tag{1.6}$$

The maximal CP violation hypothesis demands $\sin\delta_{13} = 1$, so that we obtain

$$J \simeq |V_{us}||V_{cd}||V_{ub}| \ , \tag{1.7}$$

where we have used the observed fact $1 \gg |V_{us}|^2 \gg |V_{cd}|^2 \gg |V_{ub}|^2$. The choice $\delta_{13} = \pi/2$ also predicts

$$|V_{td}| = \sqrt{(s_{23}s_{12})^2 + (c_{23}c_{12}s_{13})^2} = 0.00976 \pm 0.00016 \ , \tag{1.8}$$

$$\alpha = 68.5°^{+3.2°}_{-2.7°} \simeq \sin^{-1}(|V_{us}||V_{cb}|/|V_{td}|) \ , \tag{1.9}$$

$$\beta = 21.5°^{-3.2°}_{+2.7°} \simeq \sin^{-1}(|V_{ub}|/|V_{td}|) \ , \tag{1.10}$$

$$\gamma = 89.96° \pm 0.00° \simeq \sin^{-1}(1) \ , \tag{1.11}$$

where angles α, β and γ are defined by

$$\alpha = \mathrm{Arg}\left[-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right] = \sin^{-1}\left[\frac{|V_{12}||V_{22}|}{|V_{31}|(1 - |V_{13}|^2)} \sin\delta_{13}\right] \ , \tag{1.12}$$

$$\beta = \mathrm{Arg}\left[-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right] = \sin^{-1}\left[\frac{|V_{11}||V_{12}||V_{13}|}{|V_{21}||V_{31}|(1 - |V_{13}|^2)} \sin\delta_{13}\right] \ , \tag{1.13}$$

$$\gamma = \mathrm{Arg} \left[-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*} \right] = \sin^{-1} \left[\frac{|V_{12}||V_{33}|}{|V_{21}|(1-|V_{13}|^2)} \sin \delta_{13} \right] , \tag{1.14}$$

and we have used the observed values [7]

$$\begin{aligned} |V_{us}| &= 0.2200 \pm 0.0026 , \\ |V_{cb}| &= 0.0413 \pm 0.0015 , \\ |V_{ub}| &= 0.00367 \pm 0.00047 . \end{aligned} \tag{1.15}$$

The world average value of β [7] which has been obtained from B_d decays is

$$\sin 2\beta = 0.736 \pm 0.049 \quad \left(\beta = 23.7^{\circ} {}^{+2.2^{\circ}}_{-2.0^{\circ}} \right) , \tag{1.16}$$

so that the prediction (1.10) is in good agreement with the observed value. However, on the other hand, the best fit for the CKM parameters [7] gives

$$\gamma = 60^{\circ} \pm 14^{\circ} , \quad \beta = 23.4^{\circ} \pm 2^{\circ} , \tag{1.17}$$

so that the prediction of γ, (1.11), is entirely in disagreement with the experiments. Therefore, the maximal CP violation hypothesis must be ruled out.

However, note that this maximal CP violation hypothesis depends on the phase convention of the CKM matrix. If we use the original KM phase convention, the rephasing invariant quantity J is given by

$$J = c_1 s_1^2 c_2 s_2 c_3 s_3 \sin \delta_{KM} , \tag{1.18}$$

i.e.

$$J = \frac{|V_{11}||V_{12}||V_{13}||V_{21}||V_{31}|}{1 - |V_{11}|^2} \sin \delta_{SD} , \tag{1.19}$$

and the requirement $\delta_{KM} = \pi/2$ predicts

$$J \simeq |V_{ub}||V_{td}| , \tag{1.20}$$

$$|V_{ub}| = s_1 s_2 \simeq |V_{us}||V_{cb}|\sqrt{1 - \xi^2} , \tag{1.21}$$

where

$$\xi = |V_{ub}|/|V_{us}||V_{cb}| . \tag{1.22}$$

(The relations between V_{SD} and V_{KM} can, for instance, be found in Ref.[8].) From the observed values (1.15), we obtain the numerical results

$$|V_{td}| = 0.0084 \pm 0.0005, \tag{1.23}$$

$$\alpha = 89.96^{\circ} \pm 0.00^{\circ}, \tag{1.24}$$

$$\beta = 23.2^{\circ} {}^{-3.8^{\circ}}_{+3.5^{\circ}}, \tag{1.25}$$

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$$\gamma = 66.8°^{+3.8°}_{-3.5°}. \tag{1.26}$$

These results are in good agreement with the observed values (1.16) and (1.17).

Thus, the results from the maximal CP violation hypothesis depend on the phase convention. (Note that we have applied the maximal CP violation hypothesis to the CKM phase convention V_{KM}, (1.2), under the rotation parameters fixed. If we apply the hypothesis to V_{KM} under $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ fixed, the results are same as in the standard phase convention.) Such phase-convention dependence, in spite of the rephasing invariance of the CKM matrix, is due to that we tacitly assume that only the phase parameter δ_{13} (δ_{KM}) is free and it is independent of the rotation parameters s_{ij} (s_i).

In the present paper, we systematically investigate whether there is other phase convention which gives successful predictions or not, and we will find an interesting phase convention which speculates successful relations for quark masses m_{qi} and the CKM matrix elements $|V_{ij}|$.

2 Phase conventions and the expressions of J

Let us give the CKM matrix V as

$$V = V(i,k) \equiv R_i^T P_j R_j R_k \qquad (i \neq j \neq k), \tag{2.1}$$

where R_i ($i = 1, 2, 3$) are defined by Eqs. (1.3), and P_i are given by $P_1 = \mathrm{diag}(e^{i\delta},\ 1,\ 1)$, $P_2 = \mathrm{diag}(1,\ e^{i\delta},\ 1)$, and $P_3 = \mathrm{diag}(1,\ 1,\ e^{i\delta})$, we can show that the magnitudes of the CKM matrix elements, $|V_{i1}|$, $|V_{i2}|$, $|V_{i3}|$, $|V_{1k}|$, $|V_{2k}|$ and $|V_{3k}|$, do not depend on the phase parameter δ, and the rephasing invariant quantity J is given by

$$J = \frac{|V_{i1}||V_{i2}||V_{i3}||V_{1k}||V_{2k}||V_{3k}|}{(1 - |V_{ik}|^2)|V_{ik}|} \sin \delta . \tag{2.2}$$

Note that the expression (2.2) is only dependent on i and k, and it is independent of j . Therefore, we have nine cases of $V(i, k)$. (This has been pointed out by Fritzsch and Xing [9].) The expressions $V(1, 3)$ and $V(1, 1)$ correspond to the standard and original KM phase conventions, respectively.

For the observed fact $1 \gg |V_{us}|^2 \simeq |V_{cd}|^2 \gg |V_{cb}|^2 \simeq |V_{ts}|^2 \gg |V_{ub}|^2$, the results (2.2) are approximately given as follows:

$$J \simeq |V_{us}||V_{cb}||V_{ub}| \sin \delta , \tag{2.3}$$

for $V(1, 2)$, $V(1, 3)$, $V(2, 1)$ and $V(2, 3)$

$$J \simeq |V_{ub}||V_{td}| \sin \delta , \tag{2.4}$$

for $V(1, 1)$ and $V(3, 3)$

$$J \simeq |V_{cb}|^2 \sin \delta , \tag{2.5}$$

for $V(2,2)$, and

$$J \simeq |V_{us}||V_{cb}||V_{td}| \sin \delta \ , \tag{2.6}$$

for $V(3,1)$ and $V(3,2)$. The cases which can give reasonable predictions for unitary triangle under the maximal CP violation hypothesis are only the cases $V(1,1)$ and $V(3,3)$.

The explicit expression of $V(1,1)$ has already been given by Eq. (1.2). The explicit expression of $V(3,3)$ is given by

$$V(3,3) = R_3^T(\theta_{12}^u)P_1(\delta)R_1(\theta_{23})R_3(\theta_{12}^d)$$

$$= \begin{pmatrix} e^{i\delta}c_{12}^u c_{12}^d + c_{23}s_{12}^u s_{12}^d & e^{i\delta}c_{12}^u s_{12}^d - c_{23}s_{12}^u c_{12}^d & -s_{23}s_{12}^u \\ e^{i\delta}s_{12}^u c_{12}^d - c_{23}c_{12}^u s_{12}^d & e^{i\delta}s_{12}^u s_{12}^d + c_{23}c_{12}^u c_{12}^d & s_{23}c_{12}^u \\ -s_{23}s_{12}^d & -s_{23}c_{12}^d & c_{23} \end{pmatrix} , \tag{2.7}$$

which has been proposed by Fritzsch and Xing [10]. For the expression (2.7), we obtain the expression of J

$$J = c_{23}s_{23}^2 c_{12}^u s_{12}^u c_{12}^d s_{12}^d \sin \delta = \frac{|V_{13}||V_{23}||V_{33}||V_{32}||V_{31}|}{1 - |V_{33}|^2} \sin \delta \ , \tag{2.8}$$

and the relations

$$\frac{s_{12}^u}{c_{12}^u} = \frac{|V_{ub}|}{|V_{cb}|} \ , \tag{2.9}$$

$$\frac{s_{12}^d}{c_{12}^d} = \frac{|V_{td}|}{|V_{ts}|} \ , \tag{2.10}$$

$$s_{23} = \sqrt{|V_{cb}|^2 + |V_{ub}|^2} \ . \tag{2.11}$$

Under the maximal CP violation hypothesis, since the matrix element $|V_{us}|$ is given

$$|V_{us}| = \sqrt{(c_{12}^u s_{12}^d)^2 + (c_3 s_{12}^u c_{12}^d)^2} \ , \tag{2.12}$$

the value of s_{12}^d can be fixed by the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. It is approximately given by

$$s_{12}^d \simeq |V_{us}|\sqrt{1 - \xi^2} \ , \tag{2.13}$$

where ξ is defined by Eq. (1.22). When we use the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, (1.15), the numerical predictions without approximation are as follows:

$$J = (3.01_{+0.10}^{-0.22}) \times 10^{-5}, \tag{2.14}$$

$$|V_{td}| = 0.00842 \pm 0.00052, \tag{2.15}$$

$$\alpha = 88.95°^{+0.14°}_{-0.12°}, \tag{2.16}$$

$$\beta = 23.2°^{-3.8°}_{+3.5°}, \tag{2.17}$$

$$\gamma = 67.8°^{+2.7°}_{-4.4°}. \tag{2.18}$$

These numerical results are approximately same as those in the original KM phase convention, but are slightly different from the results (1.8)–(1.11).

3 Speculation on the quark mass matrix form

In the maximal CP violation hypothesis, we have, so far, assumed that the rotation parameters are fixed and only free parameter is the CP violation phase δ. This suggests the following situation. The phase factors in the quark mass matrices M_f $(f = u, d)$ are factorized by the phase matrices P_f as

$$M_f = P_{fL}^\dagger \widetilde{M}_f P_{fR} , \tag{3.1}$$

where P_f are phase matrices and $\widetilde{M}_f$ are real matrices. The real matrices $\widetilde{M}_f$ are diagonalized by rotation (orthogonal) matrices R_f as

$$R_f^\dagger \widetilde{M}_f R_f = D_f \equiv \mathrm{diag}(m_{f1},\ m_{f2},\ m_{f3}), \tag{3.2}$$

(for simplicity, we have assumed that M_f are Hermitian or symmetric matrix, i.e. $P_{fR} = P_{fL}$ or $P_{fR} = P_{fL}^\dagger$ respectively), so that the CKM matrix V is given by

$$V = R_u^T P R_d , \tag{3.3}$$

where $P = P_{uL}^\dagger P_{dL}$. The quark masses m_{fi} are only determined by $\widetilde{M}_f$. In other words, the rotation parameters are given only in terms of the quark mass ratios, and independent of the CP violating phases. In such a scenario, the maximal CP violation hypothesis means that the CP violation parameter δ takes its maximum value without changing the quark mass values.

For example, the choices of the standard and original KM phase conventions suggest the quark mass matrix structures

$$\begin{aligned}
\widetilde{M}_u &= R_2(\theta_{13}^u) R_1(\theta_{23}) D_u R_1^T(\theta_{23}) R_2^T(\theta_{13}^u) , \\
\widetilde{M}_d &= R_2(\theta_{13}^d) R_3(\theta_{12}) D_d R_3^T(\theta_{12}) R_2^T(\theta_{13}^d) ,
\end{aligned} \tag{3.4}$$

with $\theta_{13} = \theta_{13}^d - \theta_{13}^u$ and

$$\begin{aligned}
\widetilde{M}_u &= R_3(\theta_1^u) R_1(\theta_2) D_u R_1^T(\theta_2) R_2^T(\theta_1^u) , \\
\widetilde{M}_d &= R_3(\theta_1^d) R_1(\theta_3) D_d R_1^T(\theta_3) R_3^T(\theta_1^d) ,
\end{aligned} \tag{3.5}$$

with $\theta_1 = \theta_1^d - \theta_1^u$, respectively. The success of the maximal CP violation hypothesis, (1.23)–(1.26), suggest that the mass matrix structure (3.5) is preferable to the structure (3.4). However,

another candidate of V which gives the magnitude of J, (2.4), also gives successful results (2.14)–(2.18). The case $V(3,3)$ suggests the following quark mass matrix structure:

$$\widetilde{M}_u = R_1(\theta_{23}^u)R_3(\theta_{12}^u)D_u R_3^T(\theta_{12}^u)R_1^T(\theta_{23}^u) ,$$
$$\widetilde{M}_d = R_1(\theta_{23}^d)R_3(\theta_{12}^d)D_d R_3^T(\theta_{12}^d)R_1^T(\theta_{23}^d) ,$$

(3.6)

with $\delta = \delta_d - \delta_u$ and $\theta_{23} = \theta_{23}^d - \theta_{23}^u$. The mass matrix structure (3.6) is explicitly given by the form

$$\widetilde{M}_f = \begin{pmatrix} m_{f1}c_{12}^{f2} + m_{f2}s_{12}^{f2} & (m_{f2} - m_{f1})c_{12}^f s_{12}^f c_{23}^f & -(m_{f2} - m_{f1})c_{12}^f s_{12}^f s_{23}^f \\ (m_{f2} - m_{f1})c_{12}^f s_{12}^f c_{23}^f & (m_{f1}s_{12}^{f2} + m_{f2}c_{12}^{f2})c_{23}^{f2} + m_{f3}s_{23}^{f2} & (m_{f3} - m_{f2}c_{12}^{f2} - m_{f1}s_{12}^{f2})c_{23}^f s_{23}^f \\ -(m_{f2} - m_{f1})c_{12}^f s_{12}^f s_{23}^f & (m_{f3} - m_{f2}c_{12}^{f2} - m_{f1}s_{12}^{f2})c_{23}^f s_{23}^f & (m_{f1}s_{12}^{f2} + m_{f2}c_{12}^{f2})s_{23}^{f2} + m_{f3}c_{23}^{f2} \end{pmatrix} .$$

(3.7)

In the mass matrix (3.7), the ansatz $\widetilde{M}_{11}^d = 0$ leads to the well–known relation [11]

$$|V_{us}| \simeq s_{12}^d \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.22 .$$

(3.8)

On the other hand, in the mass matrix structure (3.5), there is no simple relation such as (3.8). Therefore, the mass matrix structure (3.6) [i.e. (3.7)] [and also the phase convention (2.7)] is more attractive to us compared with the alternative one (1.2) (the original KM phase convention). Furthermore, in the mass matrix (3.7), if we assume $\widetilde{M}_{11}^u = 0$ analogous to $\widetilde{M}_{11}^d = 0$, we obtain

$$\frac{s_{12}^u}{c_{12}^u} \simeq \sqrt{\frac{m_u}{m_c}} = 0.059 ,$$

(3.9)

where quark mass values [12] at $\mu = m_Z$ have been used. Compared with the experimental value of $|V_{ub}|/|V_{cb}|$

$$\frac{s_{12}^u}{c_{12}^u} = \frac{|V_{ub}|}{|V_{cb}|} = 0.089^{+0.015}_{-0.014} ,$$

(3.10)

the prediction (3.9) is slightly small. However, this discrepancy should not be taken seriously, because the present speculation on the quark mass matrices is made only for main framework of the mass matrices. The purpose of the present paper is to investigate a possible phase convention form which can give successful predictions for the shape of the unitary triangle under the maximal CP violation hypothesis, and not to find a phenomenologically successful quark mass matrix form, we do not go into the phenomenology of the mass matrix form (3.7) any more.

4 Conclusion

The predictions from the maximal CP violation hypothesis depend on the phase conventions of the CKM matrix V. We have systematically investigated whether the hypothesis can give successful predictions for the magnitude of the rephasing invariant quantity J and the shape of the unitary triangle or not. In conclusion, we have found that, of the nine possible phase conventions $V(i,\ k) = R_i^T P_j R_j R_k$, only two, $V(1,1)$ (the original KM phase convention) and $V(3,3)$ (the Fritzsh–Xing phase convention), can yield successful predictions.

Furthermore, we have speculated possible quark matrix forms which are suggested from the expressions $V(i,\ k)$. Since a texture-zero requirement $M_{11}^d = 0$ in the mass matrix form (3.7) can lead to the well–known relation $|V_{us}| \simeq m_d/m_s$, the new phase convention $V(3,3)$ is very attractive to us rather than the original KM phase convention $V(1,1)$. (Of course, for experimental data analysis, the standard phase convention $V(1,3)$ [i.e. (1.1)] is the most useful expression. Only for discussing the relations between the CKM matrix and the quark mass matrix forms M_f, the expression $V(3,3)$ [i.e. (2.7)] will be useful.)

Of course, we cannot ruled out a possibility that the maximal CP violation hypothesis is not true. Then, from the view point of a simple texture-zero ansatz, the phase convention $V(2,3)$ is also attractive to us, because the case suggests the quark mass matrix structure $\widetilde{M_u} = R_1^u R_2^u D_u R_2^{uT} R_1^{uT}$ and $\widetilde{M_d} = R_1^d R_3^d D_d R_3^{uT} R_1^{uT}$. The texture-zero requirements $\widetilde{M_{11}^u} = 0$ and $\widetilde{M_{11}^d} = 0$ predicts $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036$ and $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.22$, respectively. Those predictions are in good agreement with the observed values (1.15).

If we apply the mass matrix structure (3.7) to the lepton sector, we obtain

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}} = 0.049, \tag{4.1}$$

for the $V(3,3)$ model, while

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\tau}} = 0.012, \tag{4.2}$$

for the $V(2,3)$ model, where we have taken $s_{23} = c_{23} = 1/\sqrt{2}$ from the observed fact [13, 14] $\sin^2 2\theta_{atm} \simeq 1$. If a near future experiment confirms the relation (4.1), the $V(3,3)$ model which is suggested from the maximal CP violation hypothesis will become promising.

Acknowledgment

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